

Maximum lifetime problem in sensor networks with limited channel capacity

Z. Lipiński

Institute of Mathematics and Informatics
Opole University, Poland

November 2, 2015

Abstract

In the paper we analyze the maximum lifetime problem in sensor networks with limited channel capacity for multipoint-to-multipoint and broadcast data transmission services. We show, that in order to achieve an optimal data transmission regarding the maximum lifetime problem we cannot allow for any interference of signals. We propose a new Signal to Interference plus Noise Ratio function and used is to modify the Shannon-Hartley channel capacity formula. For the modified channel capacity formula we solve the maximum lifetime problem in one dimensional regular sensor network L_N for discussed data transmission services.

Key words: wireless communication, sensor network lifetime, channel capacity.

1 Introduction

Characteristic feature of sensor networks is that these consist of small electronic devices with limited power and computational resources. Most of the energy the nodes of the sensor network possess is being utilized in the process of data transmission. The energy consumption depends on the size of the network, amount of data transmitted over the network and also on the presence of noise and interference of signals within the transmission channels. On the other hand, from the network a long term operating time is expected, that the functional lifetime and services delivered by the network can be available for a long time. It seems that, one of the most important problem in sensor networks is to optimize the energy consumption of each node to extend their operating time and thus the lifetime of the whole network. We define a sensor network lifetime as the time until the first node of the network runs out of energy, [1, 2]. If each node of the network has a battery with an initial energy E_0 , then by finding the optimal energy utilization of each node we can determine the number of cycles $N_{\text{cycles}} = \lfloor \frac{E_0}{E_i^{\text{opt}}} \rfloor$ the network can perform its functions until the most overloaded node runs out of its energy E_i^{opt} .

The energy consumption model in wireless sensor networks can be expressed in terms of power P_i and the data transmission time t_i of each sensor. For the optimization problems in which we minimize the energy consumed by each node

$$E_i = P_i t_i, \quad (1)$$

we must assume that capacities of the transmission channels are limited, otherwise the minimum energy of each node is reached for $t_i = 0$. The capacity of a noisy channel can be described by the well know Shannon-Hartley formula, [3],

$$C = B \log(1 + \frac{P}{N_o}), \quad [b/s]. \quad (2)$$

where B is the channel bandwidth, P the signal and N_o the noise power. An impact on the channel capacity has not only the level of a noise but also the interference of signals generated simultaneously by nodes in a given network. To take into account the interference in wireless network the quotient $\frac{P}{N_o}$ in the Shannon-Hartley formula (2) should be replaced by a Signal to Interference plus Noise Ratio function. Let us denote by S_N the wireless sensor network built of N nodes. We assume, that the nodes are located at the points x_i of d -dimensional space R^d . By $P(x_i)$ we denote the transmission power of the i -th node and by $\gamma(x_i, x_j)$ the signal gain function between two nodes located at the points x_i and x_j . Because of the loss factor $\gamma(x_i, x_j)$ the strength of signal of the i -th node detected by the j -th node is equal to $P(x_i)\gamma(x_i, x_j)$. The Signal to Interference plus Noise Ratio (SINR) function defined in [4], see also [3, 5, 6], is given by the formula

$$s(x_i, x_j, U_i) = \frac{P(x_i)\gamma(x_i, x_j)}{N_o + \sum_{k \in U_i} P(x_k)\gamma(x_k, x_j)}, \quad (3)$$

where $U_i \subset S_N$ is the set of nodes which signals interfere with the signal of given i -th node. By definition $i \notin U_i$ for any $i \in S_N$. Note that, when the nodes use the omnidirectional antennas and their transmission power is such that the signal is detected by any other node of the network, then U_i is the set of nodes which simultaneously transmit data with the i -th node. The formula (3) allows to estimate the conditions under which the transmission from the i -th node is successfully received by the j -th node. In literature a typical threshold $\beta \leq s(x_i, x_j, U_i)$ below which the data cannot be transmitted has value of $\beta \geq 1.1$. From the formula (3) it follows that the parameter β limits the number of nodes in the set U_i . The modified Shannon-Hartley formula can be written in the form

$$C(x_i, x_j, U_i) = \log(1 + s(x_i, x_j, U_i)), \quad (4)$$

where $s(x_i, x_j, U_i)$ is the SINR function (3). In (4) the channel bandwidth B we put equal to one. The formula (4) allows to determine the maximum achievable transmission rate between the i -th and j -th node in the presence of other transmitters. If the sensors utilize the point-to-point data transmission in the physical layer then the amount of data $q_{i,j}$ transmitted between i -th and j -th node is a product of the transmission rate $c_{i,j}$ and transmission time $t_{i,j}$ of the i -th node

$$q_{i,j} = c_{i,j} t_{i,j}. \quad (5)$$

In optimization problems discussed in this paper the transmission rate matrix elements $c_{i,j}$ used as parameters satisfy the inequality

$$0 \leq c_{i,j} \leq C(x_i, x_j, U_i), \quad (6)$$

where $C(x_i, x_j, U_i)$ is given by (4).

Another model of energy consumption in wireless networks can be described in terms of data transmission cost energy matrix $E_{i,j}$ and the data flow matrix $q_{i,j}$, [1, 7]. Elements of the matrix $E_{i,j}$ define the energy required to transmit one unit of data between two nodes. The energy necessary to transmit the amount of data $q_{i,j}$ from the i -th node to the j -th node is a product $E_{i,j}q_{i,j}$ and the total the energy E_i consumed by the i -th node to transmit all of its data is given by

$$E_i = \sum_j q_{i,j} E_{i,j}. \quad (7)$$

In the paper we discuss the relation between these two energy consumption models (1) and (7). We show, that for a modified SINR function (3), such that the signal of the transmitting node depends on the distance to the receiver, the solution of the maximum lifetime problem for both energy consumption models coincide. We also show, that for a sufficiently large network the solution of the maximum lifetime problem with the SINR function given by (3) can be reduced to the solution

of the problem discussed in the papers [1, 7, 8, 9], where the energy consumption model (7) was utilized.

In this paper we consider two types of the maximum lifetime problems in sensor networks with limited channel capacity and in the presence of noise and interference of signals. The first problem is related to the optimization of the data transmission from a given set of sensors to the set of data collectors in S_N . This type of problem we call later on a multipoint-to-multipoint data transmission service. The second problem describes the maximum lifetime problem for the broadcast data transmission. The objective function of the maximum lifetime problem for both types of services has the form

$$E_{i'} = \max_{i \in S_N} \{E_i\}, \quad (8)$$

where E_i is the energy consumed by the i -th node of the network S_N . This formula can be utilized for the energy consumption model (1) and (7). In the physical layer the nodes of wireless network can use the point-to-point or the point-to-multipoint data transmission. For the point-to-point transmission the sender transmits data to the unique receiver. For the point-to-multipoint data transmission the transmitter sends parallel the same data to a set of receivers. An important feature of the point-to-multipoint data transmission is the 'wireless multicast advantage' property, [10]. For such transmission the nodes, which are in the range of the transmitting node, can receive the data without additional cost of the transmitter. In this paper we consider the maximum lifetime problems in sensor networks which nodes use in the physical layer the point-to-point data transmission.

For the multipoint-to-multipoint data transmission service the minimum of the objective function (8) is determined under the data flow transmission constraint

$$\sum_i q_{i,j} = Q_i + \sum_j q_{j,i}. \quad (9)$$

This equation states that the amount of data generated by the i -th node Q_i and the amount of data received from other nodes $\sum_j q_{j,i}$ must be equal to the amount of data which the node can send $\sum_i q_{i,j}$. By definition the data collectors does not generate, transmit or retransmit any data, i.e., for a data collector $k \in S_N$, $\forall_{j \in S_N} q_{k,j} = 0$.

For the broadcast data transmission service, when the k -th node sends the amount Q_k of data to all other nodes of the network, the requirement that each node of S_N receives this data can be written in the form

$$\forall_{j \in S_N, j \neq k} \sum_{i, i \neq j} q_{i,j} = Q_k. \quad (10)$$

In this paper we show that the optimal transmission for the maximum lifetime problem with limited channel capacity is achieved when there is no interference in the transmission channels. For models in which the nodes use the omnidirectional antennas and the threshold β is equal to zero, the requirement that there is no interference means that the data is transmitted by the nodes sequentially. We propose a new point-to-point data transmission model with SINR function in which the signal power of the transmitting node $P(x_i)$ depends on the distance to the receiver and has the form $P(x_i) = P_0 \gamma(x_i, x_j)$, where $\gamma(x_i, x_j)$ is the signal gain function. We show, that for such model the solutions of the maximum lifetime problem for both energy consumption models (1) and (7) can be written in the same form and thus are equivalent. We give the solution of the maximum lifetime problem with the new SINR function in the one dimensional regular sensor network L_N for both the multipoint-to-multipoint and broadcast data transmission services and show that these solutions coincide with the analytical solution of the problems discussed in [1, 8, 9]. We also show, that when the distance between sensor nodes is sufficiently large, the solution of the maximum network lifetime problem with Gupta-Kumar SINR function (3) coincides with the solution of the problem written in terms of $E_{i,j}$ and $q_{i,j}$ matrices discussed in [1, 8, 9].

2 Data transmission model with the SINR function

We assume, that the power of the transmitting signal at the receiver must have some minimal level P_0 . In other words, P_0 is the minimal signal power the receiver node can hear. This requirement means, that the transmitting node must generate the signal with the strength

$$P_{i,j} = P_0 \gamma_{i,j}^{-1}, \quad (11)$$

where $\gamma_{i,j} = \gamma(x_i, x_j)$ is a signal gain function between two nodes located at the points x_i and x_j of R^d . The energy required to send data between the i -th and j -th node in the time $t_{i,j}$ equals

$$E_{i,j} = P_0 \gamma_{i,j}^{-1} t_{i,j}. \quad (12)$$

When sensors use in the physical layer the point-to-point data transmission the total energy consumed by the i -th node is given by the formula

$$E_i(t) = P_0 \sum_{j \in S_N} \gamma_{i,j}^{-1} t_{i,j}. \quad (13)$$

Because the total operating time of the i -th sensor is given by the sum $\sum_{j \in S_N} t_{i,j}$ over all network nodes, the assumption that sensors use the point-to-point data transmission in the physical layer is important. We denote by $U \subset S_N \times S_N$ the set of transmitter-receiver pairs (i, k) , $i, k \in S_N$ such that for two elements (i, k) and (i', k') of U the nodes i and i' transmit data to the k and k' nodes respectively and their signals interfere. We assume, that $i \neq i'$, $k \neq k'$. For wireless networks in which the nodes use the omnidirectional antennas and the signal is received in the whole network U can be defined as a set of node pairs which transmitters transmit data simultaneously, i.e.,

$$U = \{(i, k), (i', k') \in S_N \times S_N | t_{i,k}^{(s)} = t_{i',k'}^{(s)}, t_{i,k}^{(e)} = t_{i',k'}^{(e)}\}, \quad (14)$$

where $t_{i,k}^{(s)}$ and $t_{i,k}^{(e)}$ is the start and the end of transmission time between i -th and k -th node. If $(i, k) \in U$ then by $U_{i,k}$ we denote the set $U_{i,k} = U \setminus \{(i, k)\}$. We assume, that each node cannot receive data simultaneously from many transmitters and cannot send data to several receivers, i.e. there is no wireless multicast advantage. For the transmission model (11) the maximum achievable transmission rate between the i -th and j -th nodes can be given by the formula

$$C(x_i, x_j, U_{i,j}^n) = \log\left(1 + \frac{P_0}{N_o + P_0 \sum_{(k,m) \in U_{i,j}^n} \gamma(x_k, x_m)^{-1} \gamma(x_k, x_j)}\right), \quad (15)$$

where $U_{i,j}^n$ is a given set of transmitter-receiver pairs which signal of the transmitters interfere with the signal of the i -th node.

In the next sections we show, that the solutions of the maximum network lifetime problem with the channel capacity (15) for the multipoint-to-multipoint and the broadcast data transmission services for both energy consumption models (1) and (7) are equivalent. We show that, the solution of the maximum network lifetime problem with maximum transmission rate (15) for both types of services in the one dimensional regular sensor network L_N can be obtained from the analytical solutions of the problems given in [1, 8, 9].

3 Multipoint-to-multipoint data transmission service

The maximum network lifetime problem for the multipoint-to-multipoint data transmission service with the channel capacity given by (15) is defined by the objective function (8) in which the energy E_i of each node is given by the formula

$$E_i(t) = P_0 \sum_{j,n} \gamma_{i,j}^{-1} t_{i,j}^n, \quad (16)$$

where $t_{i,j}^n$ is the data transmission time between the i -th and j -th nodes in the presence of other transmitters from the set $U_{i,j}^n$. The data flow transmission constraint (9) can be written in the form

$$\begin{cases} \sum_{i,n} c_{i,j}^n t_{i,j}^n = Q_i + \sum_{j,n} c_{j,i}^n t_{j,i}^n, \\ 0 \leq c_{i,j}^n \leq C_{i,j}^n, \end{cases} \quad (17)$$

where $C_{i,j}^n$ is the maximum transmission rate between i -th and j -th nodes calculated from (15). For the maximum network lifetime problem of the multipoint-to-multipoint data transmission service with limited channel capacity we deduct the following.

Theorem 1. The optimal data transmission for the maximum network lifetime problem (8), (16), (17) is the transmission without interference.

Proof. The energy consumed by the i -th node $E_i(t) = P_0 \sum_{j,n} t_{i,j}^n \gamma_{i,j}^{-1}$ is minimal when the data transmission time $t_{i,j}^n$ is minimal. Because of the constrain (17), the time parameter $t_{i,j}^n$ reaches the minimal value when $c_{i,j}^n = C_{i,j}^n$ and the capacity $C_{i,j}^n$ of the channel (i,j) is maximal. The $C_{i,j}^n$ is maximal when the interference factor in the SINR function $s(x_i, x_j, U_i^n)$ or $s(x_i, x_j, U_{i,j}^n)$ is equal to zero. It means, that the set U_i^n in (3) and $U_{i,j}^n$ in (15) must be empty and there is no interference of signals in the network. \diamond

The result of Theorem 1 we use to redefine the maximum network lifetime problem of the multipoint-to-multipoint data transmission service to the form

$$\begin{cases} \min_t \max_i \{E_i(t)\}, \\ E_i(t) = P_0 \sum_j \gamma_{i,j}^{-1} t_{i,j}, \\ c_0 \sum_i t_{i,j} = Q_i + c_0 \sum_j t_{j,i}, \\ t_{i,j} \geq 0, \quad 0 \leq c_0 \leq C_0, \quad i, j \in [1, N], \end{cases} \quad (18)$$

where by C_0 we denoted the maximal transmission rate in (15), achieved when $U_{i,j}^n = \emptyset$, i.e.,

$$\forall_{i,j} \quad C(x_i, x_j) = \log(1 + \frac{P_0}{N_o}) = C_0. \quad (19)$$

In the next lemma we give the solution of the maximum network lifetime problem (18) for a one dimensional, regular sensor network L_N with one data collector for arbitrary signal gain function of the form

$$\gamma^{-1}(\bar{a}, \bar{\lambda}, x_i, x_j) = \sum_{n=0}^{\infty} \lambda_n d(x_i, x_j)^{a_n},$$

where $d(x_i, x_j)$ is Euclidean distance in R and $\sum_{n=0}^{\infty} \lambda_n = 1$, $\lambda_n \geq 0$, $a_n \geq 1$. We assume, that sensors of the L_N network are located at the points $x_i = i$ of a line and the data collector at the point $x_0 = 0$. For $d_{i,j} = |i - j|$ any two sensors in L_N , which distance between them is equal to $|i - j| = r$ the signal gain $\gamma_{i,j}$ can be written in the form

$$\gamma_r^{-1}(\bar{a}, \bar{\lambda}) = \sum_{n=0}^{\infty} \lambda_n r^{a_n}, \quad r \in [1, N], \quad (20)$$

where $\sum_{n=0}^{\infty} \lambda_n = 1$, $\lambda_n \geq 0$ and $a_n \geq 1$. In the following lemma we give the solution of (18) for any gain function (20).

Lemma 1. The solution of the maximum lifetime problem for the multipoint-to-multipoint data transmission service in the one dimensional regular sensor network L_N , with the channel gain function (20) and the channel capacity (15) is given by the following set of data transmission trees

$$T^1 = T_{1,0}, \quad T^i = \{T_{i,0}^i, T_{i,i-1}^i\}, \quad i \in [2, N],$$

and weights

$$\left\{ \begin{array}{l} t_{1,0} = \frac{1}{C_0} Q_N + \frac{1}{C_0} \sum_{j=2}^N Q_{j-1} \prod_{r=j}^N (1 - \frac{\gamma_1^{-1}}{\gamma_r^{-1}}), \\ t_{2,0} = \frac{1}{C_0} \frac{\gamma_1^{-1}}{\gamma_2^{-1}} Q_1, \\ t_{i,0} = \frac{1}{C_0} \frac{\gamma_1^{-1}}{\gamma_i^{-1}} (Q_{i-1} + \sum_{j=1}^{i-2} Q_{i-j-1} \prod_{r=1}^j (1 - \frac{\gamma_1^{-1}}{\gamma_{i-r}^{-1}})), \quad i \in [3, N], \\ t_{2,1} = \frac{1}{C_0} Q_2 - \frac{1}{C_0} \frac{\gamma_1^{-1}}{\gamma_2^{-1}} Q_1, \quad N = 2, \\ t_{2,1} = \frac{1}{C_0} \sum_{k=2}^N Q_k - \frac{1}{C_0} \sum_{k=2}^N \frac{\gamma_1^{-1}}{\gamma_k^{-1}} Q_{k-1} - \frac{1}{C_0} \sum_{k=3}^N \frac{\gamma_1^{-1}}{\gamma_k^{-1}} \sum_{j=1}^{k-2} Q_{k-j-1} \prod_{r=1}^j (1 - \frac{\gamma_1^{-1}}{\gamma_{k-r}^{-1}}), \quad N \geq 3, \\ t_{i,i-1} = \frac{1}{C_0} \sum_{k=i}^N Q_k - \frac{1}{C_0} \sum_{k=i}^N \frac{\gamma_1^{-1}}{\gamma_k^{-1}} (Q_{k-1} + \frac{1}{C_0} \sum_{j=1}^{k-2} Q_{k-j-1} \prod_{r=1}^j (1 - \frac{\gamma_1^{-1}}{\gamma_{k-r}^{-1}})), \quad i \in [3, N], \end{array} \right. \quad (21)$$

Proof. It is easy to show that the formulas given in (18) by a simple transformation $q_{i,j} \rightarrow P_0 t_{i,j}$ and $E_{i,j} \rightarrow \gamma_{i,j}^{-1}$ and $Q_i \rightarrow \frac{P_0}{C_0} Q_i$ can be obtained from the formulas (8) from [8], and the solution (21) can be obtained by these transformation from the solution (16) given in [8]. Detailed proof that the set of trees and weight polynomials (21) in the Lemma 1 is optimal can be carried out analogously to the proof given in [8]. \diamond

4 Broadcast data transmission service

To define the maximum network lifetime problem for the broadcast data transmission we represent the sensor network S_N as a directed, weighted graph $G_N = \{S_N, V, E\}$, in which S_N is the set of graph nodes, V is the set edges and E the set of weights. Each directed edge $T_{i,j} \in V$ defines a communication link between i -th and j -th node of the network. To each edge $T_{i,j}$ we assign a weight $E_{i,j}$, which is the cost of transmission of one unit of data between i -th and j -th node. By $U_i^{(\text{out})} \subseteq S_N$ we denote a set of the network nodes to which the i -th node can sent the data $U_i^{(\text{out})} = \{x_j \in S_N \mid \exists T_{i,j} \in V\}$. The set $U_i^{(\text{out})}$ defines the maximal transmission range of the i -th node. We assume, that each node of S_N can send data to any other nodes of the network, i.e.,

$$\forall i \in [1, N], \quad U_i^{(\text{out})} = S_N. \quad (22)$$

We describe the data flow in the network S_N in terms of spanning trees of the graph G_N . The set of all spanning trees $T^{k,r}$, which begin in the k -th node we denote by V_k . If the assumption (22) is satisfied, then G_N is a complete graph and the number of spanning trees rooted at the k -th node $|V_k|$ is equal to N^{N-2} , [11]. By $q_{i,j}^{k,r}$ we denote the amount of data which is transmitted along the edge $T_{i,j}^{k,r}$. The amount of data $q_{i,j}^k$ send by the i -th node to the j -th node along all trees is given by the formula

$$q_{i,j}^k = \sum_r q_{i,j}^{k,r} T_{i,j}^{k,r}. \quad (23)$$

We require, that along each tree the transmitted data is the same. This means that, for fixed k and r , the weights $q_{i,j}^{k,r}$ of the edges $T_{i,j}^{k,r}$ are equal. We denote these weights by q_r^k , i.e.,

$$\forall i,j \quad q_{i,j}^{k,r} = q_r^k. \quad (24)$$

In the energy consumption model (1), when the data is transmitted without interference, the parameter q_r^k will be replaced by the parameter t_r^k which defines the transmission time for the amount q_r^k of data along the r -th tree. Because we assumed, that all data for the broadcast data transmission service is transmitted along some set of trees, we must modify in the channel capacity formula (15) the definition of the set $U_{i,j}^n$. We replace it by $U_{i,j}^{k,n,r}$, the set of sensors which signal

interferes with the signal of the i -th node when the broadcasted data is transmitted along the tree $T^{k,r}$. We denote the new channel capacity function by $C(x_i, x_j, U_{i,j}^{k,n,r})$.

The maximum network lifetime problem of the broadcast data transmission service with the channel capacity (15) with the set $U_{i,j}^{k,n,r}$ is defined by the objective function (8) in which the energy of each node is given by the formula

$$E_i(t^k) = P_0 \sum_{n,r,j} \gamma_{i,j}^{-1} t_{i,j}^{k,n,r} T_{i,j}^{k,r}, \quad (25)$$

where $\gamma_{i,j}^{-1}$ is given by (20). The data flow transmission constraint (10), which is the requirement that each node of the network must receive data broadcasted by the k -th node, can be written in the form

$$\begin{cases} \sum_{n,r,i} c_{i,j}^{k,n,r} t_{i,j}^{k,n,r} T_{i,j}^{k,r} = Q_k, & j \in [1, N], \\ t_{i,j}^{k,n,r} \geq 0, & 0 \leq c_{i,j}^{k,n,r} \leq C_{i,j}^{k,n,r}, \quad T^{k,r} \in V_k, \quad r \in [1, N^{N-2}], \end{cases} \quad (26)$$

where $C(x_i, x_j, U_{i,j}^{k,n,r}) \equiv C_{i,j}^{k,n,r}$ is given by (15). For the maximum network lifetime problem of the broadcast data transmission service defined by (8), (25), (26) we propose a theorem, similar to the Theorem 1 for the multipoint-to-multipoint data transmission service.

Theorem 2. The optimal data transmission for the maximum network lifetime broadcasting problem (8), (25), (26) is the transmission without interference.

Proof. The proof is analogous to the proof of Theorem 1. \diamond .

From the Theorem 2 it follows that $\forall_{n,i,j} C_{i,j}^{k,n,r} = C_0$ and C_0 is given by (19). Because the transmission time between i -th and j -th node, when the data is transmitted along the edge $T_{i,j}^{k,n,r}$ must be the same then we have the following relation

$$\forall_{n,i,j} t_{i,j}^{k,n,r} = t_r^k.$$

The formula (5) written in terms of trees for each set $U_{i,j}^{k,n,r}$ has the form

$$\forall_{(i,j) \in T^{k,r}} q_r^k = c_{i,j}^{k,n,r} t_{i,j}^{k,n,r}.$$

The maximum network lifetime problem of the broadcast data transmission service without interference can be written in the form

$$\begin{cases} \min_{t^k} \max_{i \in S_N} \{E_i(t^k)\}_{i=1}^N, & k \in [1, N], \\ E_i(t^k) = P_0 \sum_{j,r} \gamma_{i,j}^{-1} t_r^k T_{i,j}^{k,r}, \\ c_0 \sum_{i,r} t_r^k T_{i,j}^{k,r} = Q_k, & j \in [1, N], \\ t_r^k \geq 0, & 0 \leq c_0 \leq C_0, \quad T^{k,r} \in V_k, \quad r \in [1, N^{N-2}]. \end{cases} \quad (27)$$

The following lemma gives the solution of the problem (27) for the internal nodes of the one dimensional regular sensor network L_N .

Lemma 2. The solution of the maximum network lifetime problem for the broadcast data transmission service (27) in the one dimensional regular sensor network L_N with the channel gain function (20) and channel capacity (19) is given by the following set of data transmissions trees T^r

$$\begin{aligned} & (T_{k,k-1}^r, \dots, T_{2,1}^r, T_{r,k+1}^r, T_{k+1,k+2}^r, \dots, T_{N-1,N}^r), & r \in [1, k-1], \\ & (T_{k,k-1}^k, \dots, T_{2,1}^k, T_{k,k+1}^k, \dots, T_{N-1,N}^k), & r = k, \\ & (T_{k,k+1}^r, \dots, T_{N-1,N}^r, T_{r,k-1}^r, T_{k-1,k-2}^r, \dots, T_{2,1}^r), & r \in [k+1, N], \end{aligned} \quad (28)$$

and weights t_r^k

$$t_r^k = \begin{cases} \frac{\gamma_1^{-1}}{\gamma_k^{-1}} t_k^k + \frac{\gamma_1^{-1}}{\gamma_k^{-1}} \frac{1}{C_0} Q_k, & r = 1, \\ \frac{\gamma_1^{-1}}{\gamma_{k+1-r}^{-1}} t_k^k, & r \in [2, k-1], k \geq 3, \\ \frac{\gamma_1^{-1}}{\gamma_{r-k+1}^{-1}} t_k^k, & r \in [k+1, N-1], k \leq N-2, \\ \frac{\gamma_1^{-1}}{\gamma_{N-k+1}^{-1}} t_k^k + \frac{\gamma_1^{-1}}{\gamma_{N-k+1}^{-1}} \frac{1}{C_0} Q_k, & r = N, \end{cases} \quad (29)$$

where

$$t_k^k = \frac{1 - \frac{\gamma_1^{-1}}{\gamma_k^{-1}} - \frac{\gamma_1^{-1}}{\gamma_{N-k+1}^{-1}}}{-1 + \sum_{i=1}^k \frac{\gamma_1^{-1}}{\gamma_i^{-1}} + \sum_{i=1}^{N-k+1} \frac{\gamma_1^{-1}}{\gamma_i^{-1}}} \frac{1}{C_0} Q_k. \quad (30)$$

and γ_r^{-1} , $r \in [1, N]$ is given by (20).

Proof of this lemma can be found in [9]. For the boundary nodes $i = 1, N$ of L_N the solution of (27) is trivial. Proof of this fact can be found in [9].

5 Gupta-Kumar model without interference

We apply the results of Theorem 1 and Theorem 2 to the maximum network lifetime problem with channel capacity function given by (3). If nodes of the network transmit data with the some constant power $P(x_i) = P_0$ and there is no interference, then the maximum transmission rate in (3) is given by the expression

$$C(x_i, x_j) = \log(1 + \frac{P_0}{N_o} \gamma(x_i, x_j)). \quad (31)$$

From (31) it follows that even when there is no interference the function $C(x_i, x_j)$ still depends on the distance between transmitter and receiver. In models, in which the SINR function (3) is utilized, the nodes of the sensor network do not adjust the transmission power to the distance of the receiver. Thus, we must modify the formulas for the energy of the node (16) and (25). The maximum network lifetime problem of the multipoint-to-multipoint transmission service with the channel capacity (31) has the form

$$\begin{cases} \min_t \max_i \{E_i(t)\}, \\ E_i(t) = P_0 \sum_j t_{i,j}, \\ \sum_i c_{i,j} t_{i,j} = Q_i + \sum_j c_{j,i} t_{j,i}, \\ t_{i,j} \geq 0, 0 \leq c_{i,j} \leq C_{i,j}, \quad i, j \in [1, N], \end{cases} \quad (32)$$

where $C_{i,j} \equiv C(x_i, x_j)$ is given by (31). In (32) we put $c_{i,j} = C_{i,j}$ and rescaled the time variables as follows $t_{i,j} \rightarrow \frac{C_0}{C_{i,j}} t_{i,j}$, where C_0 is an arbitrary positive number of dimension $[b/s]$. Because $\log(1+x) \approx x$ for $|x| < 1$, then for $\frac{P_0}{N_o} \gamma(x_i, x_j) < 1$ we can write

$$C_{i,j} = \log(1 + \frac{P_0}{N_o} \gamma(x_i, x_j)) \approx \frac{P_0}{N_o} \gamma(x_i, x_j).$$

This means, that the maximum lifetime problem for the multipoint-to-multipoint transmission service can be transformed to the form (18), where $E_i(t) = N_o C_0 \sum_j \gamma_{i,j}^{-1} t_{i,j}$ and $c_0 = C_0$. The requirement $\frac{P_0}{N_o} \gamma(x_i, x_j) < 1$, for $\gamma_{i,j} = (d_{i,j})^{-a}$ can be written as $\frac{P_0}{N_o} < (d_{i,j})^a$, which means that for a sufficiently large network the two models (3), (4) and (15) coincide.

Similar results can be obtained for the broadcast data transmission service with the channel capacity function (3). For the broadcast data transmission service the third formula in (32) should be replaced by the third formula in (27) and the energy of each node by $E_i(t^k) = P_0 \sum_{j,r} t_r^k T_{i,j}^{k,r}$.

6 Conclusions

In the paper we analyzed the maximum network lifetime problem in sensor networks with limited channel capacity for multipoint-to-multipoint and broadcast data transmission services. We showed, that for the optimal data transmission of the maximum lifetime problem there is no interference of signals in the network. We introduced new Signal to Interference plus Noise Ratio function which was defined under the assumption that the signal power $P(x_i)$ of the transmitting node depends on the signal gain $\gamma(x_i, x_j)$ and location of the receiver $P(x_i) = P_0\gamma(x_i, x_j)$. For such transmission model with the channel capacity defined by the new SINR function we solved the maximum lifetime problem in one dimensional regular sensor network L_N for both data transmission services. We showed, that these solutions coincide with the solutions of the maximum lifetime problem analyzed in [1, 7, 8, 9], where the problem was defined in terms of data transmission cost energy matrix $E_{i,j}$ and data flow matrix $q_{i,j}$. This way we find the relation between the energy consumption model written in terms of $E_{i,j}$ and $q_{i,j}$ and the model defined in terms of signal power, transmission time and channels capacities.

There are several differences between the SINR functions $s(x_i, x_j, U_{i,j}^n)$ defined in (15) and proposed by Gupta and Kumar in (3). It seems that, the most interesting future of the proposed SINR functions is that for a gain function of the form $\gamma(x_k, x_m) = d(x_k, x_m)^{-a}$ the formula (15) is invariant under the rescaling of the distance between network nodes $d'(x_k, x_m) = \lambda d(x_k, x_m)$. From which follows, that the solutions of the maximum lifetime problem for two networks with proportional distance between nodes are related by a constant factor. For example, when we extend the distance between all nodes by the same factor $d(x_k, x_m) \rightarrow \lambda d(x_k, x_m)$, and the node energy for the distance $d(x_k, x_m)$ is equal $E_i(t) = P_0 \sum_{j,n} d_{i,j}^a t_{i,j}^n$, then the nodes must increase their power signal and the energy consumption increases to $E_i(t) = \lambda^a P_0 \sum_{j,n} d_{i,j}^a t_{i,j}^n$.

References

- [1] A. Giridhar, P.R. Kumar, Maximizing the functional lifetime of sensor networks. In IPSN 05: Proceedings of the 4-th International Symposium on Information Processing in Sensor Networks, Piscataway, NJ, USA, 2005. IEEE Press.
- [2] T. Acharya, G. Paul, Maximum lifetime broadcast communications in cooperative multihop wireless ad hoc networks: Centralized and distributed approaches, Ad Hoc Networks 11 (2013), pp.1667-1682.
- [3] M. Franceschetti, R. Meester, Random Networks for Communication, Cambridge University Press, 2007.
- [4] P. Gupta, P. R. Kumar, The Capacity of Wireless Networks, IEEE Transactions on Information Theory, 46(2), pp.388-404, 2000.
- [5] F. Baccelli, B. Blaszczyzyn, Stochastic Geometry and Wireless Networks, vol. 1, 2, Now Publishers Inc, 2009.
- [6] M. Grossglauser, D. Tse, Mobility Increases the Capacity of Ad-hoc Wireless Networks, IEEE/ACM Transactions on Networking, 2002, vol 10, issue 4, pp.477 - 486.
- [7] J.H. Chang, L. Tassiulas, Energy Conserving Routing in Wireless Ad-hoc Networks, Proceedings INFOCOM 2000, pp.22-31.
- [8] Z. Lipiński, Stability of routing strategies for the maximum lifetime problem in ad-hoc wireless networks, <http://arxiv.org/abs/1407.3646>

- [9] Z. Lipiński, Maximum lifetime broadcasting problem in sensor networks, <http://arxiv.org/>
- [10] J. E. Wieselthier, G. D. Nguyen, A. Ephremides, Algorithms for Energy-Efficient Multicasting in Static Ad Hoc Wireless Networks, *Mobile Networks and Applications* 6, pp. 251-263, 2001.
- [11] C. Berge, *Graphs*, North Holland, 1989.